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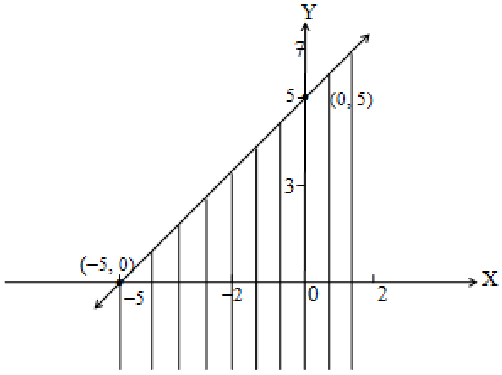

Figure (ix)

Acknowledgements

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Trilinear and Quadrilinear Forms

Wai Wai Tun[1], Aye Aye Maw[2]

Abstract

Most of the partial differential equations that arise in Continuum Mechanics and Physics are nonlinear. Because of their nonlinearity, the mathematical study of these equations is difficult and require the full power of modern functional analysis. This paper deals with the trilinear and quadrilinear forms to construct the variational formulation of some nonlinear partial differential equations of higher order.

Key words: trilinear, quadrilinear, variational formulation

1. Introduction

Let Ω be a Lipschitz open bounded subset in $\mathbb{R}^n$. We shall use the notation of the spaces V={$u\in$ D (Ω), *div* $u = 0$}, $\boldsymbol{V}$ = the closure of V in $H_0^1(\Omega), H$ = the closure of V in $L^2(\Omega)$, W = D (Ω), W = the closure of W in $H_0^1(\Omega), G$ = the closure of W in $L^2(\Omega)$. Let V', W', H' and G' denote the dual spaces of *V, W, H* and *G*. Then we have the inclusions $V \subseteq H \equiv H' \subseteq V'$ and $W \subseteq G \equiv G' \subseteq W'$.

1.1 Lemma [Temam, R. 1977] *Let V, H,* V' *be three Hilbert spaces with* $V \subseteq H \equiv H' \subseteq V'$. Let $u \in L^2(0,T;V)$ and $u' \in L^2(0,T;V')$. *Then* $u:[0,T] \to H$ *is continuous a.e and*

$$\frac{d}{dt}|u|^2 = 2\langle u', u\rangle$$

holds in scalar distribution sense on (0, *T*).

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1.2 Lemma [Temam, R. 1977]*If* u_μ *converges to u and* u_μ *converges to v in* $L^2(0,T;V)$ *weakly and* $L^2(0,T;H)$ *strongly, then for any vector function w with components in* $C^1(\bar{Q})$,

$$\int_0^T b\left(u_\mu(t), v_\mu(t), w(t)\right)dt \to \int_0^T b\left(u(t), v(t), w(t)\right)dt.$$

1.3 Definition [Temam, R. 1983**]** Let Ω be an open set in $\mathbb{R}^n$ and let *p* be a distribution on Ω, Then, for any $v \in V$,

$$\langle grad\ p, v\rangle = \sum_{i=1}^{n}\langle D_i p, v_i\rangle = -\sum_{i=1}^{n}\langle p, D_i v_i\rangle = \langle p, div\, v\rangle = 0.$$

1.4 Definition [Temam, R. 1983**]** For fixed *u* in *V*, the mapping $V \to \mathbb{R}, v \mapsto ((u,v))$ is linear and continuous on *V*. Then there exists an element of V', denote *Au* such that $\langle Au, v\rangle = ((u,v)), \forall v \in V.$ Then $u \to Au$ is linear and continuous and also an isomorphism from *V* to V'.

1.5 Definition [Temam, R. 1983**]** Let *X* be a Hilbert space. If *v* is a function from $\mathbb{R}$ into *X* then we denote the Fourier transform of *v* by $\hat{v}$ as

$$\hat{v}(\tau) = \int_{-\infty}^{\infty} e^{-2i\pi t\tau} v(t)\,dt \text{ and}$$

the derivative in *t* of order *r* of *v* is the inverse Fourier transform of $(2i\pi\tau)^r\,\hat{v}$ or

$$D_t^r v(\tau) = (2i\pi\tau)^r\,\hat{v}(\tau).$$

The definition is consistent with the usual definition for an integer *r*.

2. Boundness of Trilinear and Quadrilinear Forms

2.1 Lemma [Temam, R. 1977**]** *If the dimension is n =3, for any open set* $\Omega = 3,$ *then*

$$\|v\|_{L^4(\Omega)} \le 2^{1/2}\|v\|_{L^2(\Omega)}^{1/4}\|grad\, v\|_{L^2(\Omega)}^{3/4}, \forall v \in H_0^1(\Omega).$$

2.2 Lemma [Temam, R. 1977**]** *Let* Ω *be bounded or unbounded and any dimension of the space* $\mathbb{R}^n$. *Then, the form b,*

$$b(u,v,w) = \sum_{i,j=1}^{n}\int_\Omega u_i\left(D_i v_j\right)w_j\,dx$$

is defined and trilinear continuous on $\left[H_0^1(\Omega)\right]^3 \cap L^n(\Omega)$.

2.3 Lemma [Temam, R. 1983**]** *for any open se*t Ω, b *is trilinear continuous on* $\left(H_0^1(\Omega)\right)^3 \cap L^n(\Omega)$. *If* Ω *is bounded and* $n \le 4$ *then b is trilinear continuous on* $\left[H_0^1(\Omega)\right]^3$.

2.4 Lemma [Temam, R. 1983**]** *Assumed that the dimension of the space is* $n \le 4$ *and* $u, v \in L^2(0,T;V)$.

Let the function $B(u,v)$ be defined by

$$\langle u(t), v(t), w\rangle = b\left(u(t), v(t), w\right), \forall w \in V, a.e\, in\, t \in [0,T]$$

then $B(u,v) \in L^1(0,T;V')$.

2.5 Lemma [Temam, R. 1977**]** *If* u_μ *converges to u and* u_μ *converges to v in* $L^2(0,T;V)$ *weakly and* $L^2(0,T;H)$ *strongly, then for any vector function w with components in* $C^1(\bar{Q})$,

$$\int_0^T b\left(u_\mu(t), v_\mu(t), w(t)\right)dt \to \int_0^T b\left(u(t), v(t), w(t)\right)dt.$$

In particular, the trilinear form $\bar{b}$ defined by

$$\bar{b}(u,\theta,\gamma)=\sum_{i=1}^{3}\int_{\Omega}u_i(D_i\theta)\gamma\,dx$$

Is well defined and trilinear continuous on $\left[H_0^1(\Omega)\right]^3\cap\left[L^2(\Omega)\right]^3$, Ω bounded and $\Omega\subseteq\mathbb{R}^3$. $\bar{b}$ also has the same properties as b for $u\in L^2(0,T;V)$ and $\theta\in L^2(0,T;W)$.

Using these results, we prove the following theorems:

2.6 Theorem *The trilinear forms* c_1 *and* c_2

$$c_1(\theta,h,\gamma)=\sum_{i=1}^{3}\int_{\Omega}\theta(curl\,h)_i\gamma dx$$

$$c_2(h,\theta,\gamma)=\sum_{i=1}^{3}\int_{\Omega}\theta(curl\,h)_i(D_i\theta)\gamma dx$$

are defined and trilinear continuous on $\left[H_0^1(\Omega)\right]^3\cap\left[L^2(\Omega)\right]^3$, Ω *bounded in* $\mathbb{R}^3$ *for*

$$\left|(curl\,h)_i\right|\ll 1 \text{ and } (D_i\theta)\ll 1,\ i=1,2,3.$$

Proof: By general Hölder inequality

$$\left|\int_{\Omega}\theta(curl\,h)_i\,\gamma\,dx\right|\le|\theta|_{L^4(\Omega)}\left|(curl\,h)_i\right|_{L^2(\Omega)}|\gamma|_{L^4(\Omega)},$$

$$\sum_{i=1}^{3}\left|\int_{\Omega}\theta(curl\,h)_i\,\gamma\,dx\right|\le\sum_{i=1}^{3}|\theta|_{L^4(\Omega)}\left|(curl\,h)_i\right|_{L^2(\Omega)}|\gamma|_{L^4(\Omega)}. \tag{1}$$

Then $c_1(\theta,h,\gamma)$ is well-defined and

$$c_1(\theta,h,\gamma)\le K_0(\Omega)\|\theta\|_{H_0^1(\Omega)}\|h\|_{H_0^1(\Omega)}\|\gamma\|_{H_0^1(\Omega)}.$$

Therefore the form c_1 is trilinear continuous on $\left[H_0^1(\Omega)\right]^3\cap\left[L^2(\Omega)\right]^3$.

Also, by using Hölder inequality, we obtain

$$\left|\int_{\Omega}(curl\,h)(D_i\theta)\gamma\,dx\right|_i\le\int_{\Omega}\left|(curl\,h)_i(D_i\theta)\gamma\right|dx.$$

Choose $\left|\int_{\Omega}(curl\,h)_i\right|\ll 1, i=1,2,3.$

By Hölder inequality, we obtain

$$\left|\int_{\Omega}(curl\,h)(D_i\theta)\gamma\,dx\right|\le\int_{\Omega}\left|(D_i\theta)\gamma\right|dx$$

$$\le\int_{\Omega}\left|(D_i\theta)\right|_{L^2(\Omega)}$$

and then

$$\sum_{i=1}^{3}\int_{\Omega}\left|(curl\,h)_i(D_i\theta)\gamma\right|dx\le\sum_{i=1}^{3}|D_i\theta|_{L^2(\Omega)}|\gamma|_{L^2(\Omega)},$$

$$\left|c_2(h,\theta,\gamma)\right|\le\varepsilon_1\|\theta\|_{H_0^1(\Omega)}|\gamma|_{L^2(\Omega)}. \tag{2}$$

Again, we can choose $|D_i\theta|\ll 1$, by Hölder inequality,

$$\left|\int_{\Omega}(curl\,h)(D_i\theta)\gamma\,dx\right|_i\le\int_{\Omega}\left|(curl\,h)_i\,\gamma\right|dx$$

$$\le\int_{\Omega}\left|(curl\,h)_i\,\gamma\right|_{L^2(\Omega)}$$

then we get

$$\sum_{i=1}^{3}\int_{\Omega}\left|(curl\,h)_i\,(D_i\theta)\,\gamma\right|dx \le \sum_{i=1}^{3}\left|(curl\,h)_i\right|_{L^2(\Omega)}|\gamma|_{L^2(\Omega)},$$

$$\left|c_2(h,\theta,\gamma)\right| \le \varepsilon_2\,\|h\|_{H_0^1(\Omega)}|\gamma|_{L^2(\Omega)}. \qquad (3)$$

According to (2) and (3), the form c_2 is trilinear continuous on $\left[H_0^1(\Omega)\right]^3 \cap \left[L^2(\Omega)\right]^3$.

By (2) and (3), we have the inequality

$$\left|c_2(h,\theta,\gamma)\right| \le \left[\varepsilon_1\,\|\theta\|_{H_0^1(\Omega)} + \varepsilon_2\,\|h\|_{H_0^1(\Omega)+}\right]|\gamma|_{L^2(\Omega)}. \qquad (4)$$

2.7 Theorem *The form* c_3,

$$c_3(h,\theta,\alpha,\gamma) = \sum_{i=1}^{3}\int_{\Omega}(curl\,h)_i\,(D_i\theta)\,\alpha\gamma\;dx$$

is defined and quadrilinear continuous on $\left[H_0^1(\Omega)\right]^4 \cap \left[L^2(\Omega)\right]^4$, Ω *bounded subset in* $\mathbb{R}^3$ *and for* $\left|(curl\,h)_i\right| \ll 1$ *and* $\left|(D_i\theta)\right| \ll 1,\ i=1,2,3.$

Proof: From the form c_3, we have the inequality

$$\left|\int_{\Omega}(curl\,h)_i\,(D_i\theta)\,\alpha\gamma\;dx\right|_i \le \int_{\Omega}\left|(curl\,h)_i\,(D_i\theta)\,\alpha\gamma\right|\,dx.$$

Choose $\left|(curl\,h)_i\right| \ll 1, i=1,2,3$ and using general Hölder inequality,

$$\left|\int_{\Omega}(curl\,h)_i\,(D_i\theta)\,\alpha\gamma\;dx\right|_i \le |D_i\theta|_{L^2(\Omega)}\,\|\alpha\|_{L^4(\Omega)}\,\|\gamma\|_{L^4(\Omega)}.$$

$$\sum_{i=1}^{3}\left|\int_{\Omega}(curl\,h)_i\,(D_i\theta)\,\alpha\gamma\;dx\right| \le \sum_{i=1}^{3}|D_i\theta|_{L^2(\Omega)}\,\|\alpha\|_{L^4(\Omega)}\,\|\gamma\|_{L^4(\Omega)}.$$

So,

$$\left|c_3(h,\theta,\alpha,\gamma)\right| \le \varepsilon_3\,\|\theta\|_{H_0^1(\Omega)}\,\|\alpha\|_{L^4(\Omega)}\,|\gamma|_{L^4(\Omega)}. \qquad (5)$$

Again, choosing $|D_i\theta| \ll 1$, we get

$$\sum_{i=1}^{3}\left|\int_{\Omega}(curl\,h)_i\,(D_i\theta)\,\alpha\gamma dx\right| \le \sum_{i=1}^{3}\left|(curl\,h)_i\right|_{L^2(\Omega)}\|\alpha\|_{L^4(\Omega)}\,\|\gamma\|_{L^4(\Omega)}.$$

It leads to the inequality

$$\left|c_3(h,\theta,\alpha,\gamma)\right| \le \varepsilon_4\,\|h\|_{H_0^1(\Omega)}\,\|\alpha\|_{L^4(\Omega)}\,|\gamma|_{L^4(\Omega)}. \qquad (6)$$

By using (5) and (6), we can conclude $c_3(h,\theta,\alpha,\gamma)$ is well-defined and continuous. Therefore, the form is quadrilinear and continuous.

From (5) and (6), we get inequality

$$\left|c_3(h,\theta,\alpha,\gamma)\right| \le \left(\varepsilon_3\,\|\theta\|_{H_0^1(\Omega)} + \varepsilon_4\,\|h\|_{H_0^1(\Omega)}\right)\|\alpha\|_{L^4(\Omega)}\,|\gamma|_{L^4(\Omega)}. \qquad (7)$$

3. Some Properties of Trilinear and Quadrilinear Forms

3.1 Fundamental Properties of Trilinear Forms b and $\overline{b}$

(i) $b(u,v,v) = 0, \forall u \in V, v \in H_0^1(\Omega) \cap L^2(\Omega).$

(ii) $b(u,v,w) = -b(u,w,v), \forall u \in V, v,w \in H_0^1(\Omega) \cap L^2(\Omega).$

(iii) $\overline{b}(u,v,v) = 0, \forall u \in V, v \in H_0^1(\Omega) \cap L^2(\Omega).$

(iv) $\overline{b}(u,v,w) = -\overline{b}(u,w,v), \forall u \in V, v,w \in H_0^1(\Omega) \cap L^2(\Omega).$

3.2 Properties of Trilinear Form c_2 and Quadrilinear Form c_3

(i) $c_2(h,\theta,\theta)=0,$

(ii) $c_3(h,\theta,\theta,\theta)=0, \forall h\in H_0^1(\Omega).$

3.3 Lemma *Let Ω be a bounded subset of* $\mathbb{R}^3$. If $h\in L^2(0,T;V), \theta\in L^2(0,T;W),$ $\alpha\in L^2(0,T;W)$ *and* $\gamma\in W$ *and the functions* $C_1(\theta,h), C_2(h,\theta)$ *and* $C_3(h,\theta,\alpha)$ *defined by*

$$\langle C_1(\theta,h),\gamma\rangle = c_1(\theta(t),h(t),\gamma),$$

$$\langle C_2(h,\theta),\gamma\rangle = c_2(h(t),\theta(t),\gamma),$$

$$\langle C_3(h,\theta,\alpha),\gamma\rangle = c_3(h(t),\theta(t),\alpha(t),\gamma), \forall\gamma\in W,$$

Then $C_1(\theta,h), C_2(h,\theta)$ *and* $C_3(h,\theta,\alpha)\in L^1(0,T;W')$.

Proof: Since c_1 and c_2 are trilinear continuous, that is, $C_1: W\to W'$ and $C_2: W\to W'$ are continuous. Since $\theta\in L^2(0,T;W)$ *and* $h\in L^2(0,T;V)$ and then θ and h are measurable.

For almost all $t, \theta:[0,T]\to W'$ and $h\in[0,T]\to W$ are measurable.

So, $C_1(\theta,h):[0,T]\to W'$ and $C_2(h,\theta):[0,T]\to W'$ are measurable and

$$\int_0^T \|C_1(\theta,h)\|_{W'} dt \le K_0\int_0^T \|\theta\|_{H_0^1(\Omega)}\|h\|_{H_0^1(\Omega)}\, dt,$$

$$\int_0^T \|C_2(h,\theta)\|_{W'} dt \le \int_0^T \left(\varepsilon_1\|\theta\|_{H_0^1(\Omega)}+\varepsilon_2\|h\|_{H_0^1(\Omega)}\right) dt.$$

Therefore, $C_1(\theta,h)$ and $C_2(h,\theta)\in L^1(0,T;W')$.

Also, c_3 is quadrilinear continuous then $\|C_3(h,\theta,\alpha)\|\le\left(\varepsilon_3\|\theta\|_{H_0^1(\Omega)}+\varepsilon_4\|h\|_{H_0^1(\Omega)}\right)\|\alpha\|_{L^4(\Omega)}$.

This means that $C_3: W\to W'$ is continuous.

By the assumptions, h is a measurable function from $[0,T]$ to V, the scalar functions θ and α are measurable functions from $[0,T]$ to W. Hence, $C_3:[0,T]\to W'$ is measurable and

$$\int_0^T \|C_3(h(t),\theta(t),\alpha(t))\|_{W'} dt \le K_2\int_0^T \left(\varepsilon_3\|\theta\|_{H_0^1(\Omega)}+\varepsilon_4\|h\|_{H_0^1(\Omega)}\right)\|\alpha\|_{L^4(\Omega)}\, dt.$$

Therefore, $C_3(h,\theta,\alpha)\in L^1(0,T;W')$.

3.4 Theorem *If* u_μ *converges to* u *and* h_μ *converges to* h *in* $L^2(0,T;V)$ *weakly and in* $L^2(0,T;H)$ *strongly and* θ_μ *converges to* θ *in* $L^2(0,T;W)$ *weakly and* $L^2(0,T;G)$ *strongly, then for any scalar function* γ *in* $C^1(\bar{Q})$,

(i) $\int_0^T c_1(\theta_\mu,h_\mu,\gamma)\, dt \to \int_0^T c_1(\theta,h,\gamma)\, dt$ *and*

(ii) $\int_0^T c_2(h_\mu,\theta_\mu,\gamma)\, dt \to \int_0^T c_2(h,\theta,\gamma)\, dt.$

Proof is obvious by the properties of the trilinear forms.

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Prevalence and bionomics of *Aedes aegypti* (Linnaeus, 1762) larvaein high risk areasof Pazundaung Township, Yangon Region

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Abstract

Dengue viruses are actively transmitted by *Aedes aegypti* in many countries in the tropical zone throughout the world including Myanmar. The successful control of this species depends on knowledge of the biology and ecology of this mosquito vector including the development and survival in different container types. A total of 31 selected places (altogether 28 compounds of 9 Primary, 4 Middle and 4 High schools, 1 local health centers and 10 private day care centers/nurseries) were surveyed seasonally to determine the prevalence and bionomics of *Aedes aegypti* larvae in different container categories and types at selected areas of Pazundaung Township in relation to children aggregated areas from December, 2017 to September, 2018.Out of 31 selected places investigated (28 compounds), 16.13% in first survey, 61.29% in second survey and 38.71% in third survey of the places were found to be larva positive.

Keywords: *Aedes aegypti,* different container categories and types, positive premises

Introduction

Aedes aegypti is one of the world's most widely distributed mosquitoes and is of considerable medical importance as a vector of dengue and yellow fever (Service, 1992). The species is considered as the major vector of dengue, dengue haemorrhagic fever and dengue shock syndrome (DF/DHF/DSS) in many subtropical and tropical countries throughout the world. Prevention of DHF outbreaks in endemic areas is based on long-term anti-mosquito control measures particularly household and environmental sanitation with emphasis on larval source reduction. Only vector control promises permanency and a cost effective solution (Halstead, 1988).

In Myanmar, a severe outbreak of DHF occurred for the first time in Yangon in 1970. The urban areas within the Yangon City limits were more affected than the suburban townships of Yangon Division. This epidemic had an average morbidity of 51.97 per 100,000 population and affected mostly school going age groups.

Generally more DHF cases were abundant during rainy season especially in July and August. There was the highest number of cases in July (Ohn Kyi, 1985). However, the intervals between dengue outbreaks became shorter in the last two decades. High dengue cases in the rainy season correspond to the seasonal high densities of *Aedes aegypti* mosquitoes that are the vectors of DHF. Since Dengue/DHF is a mosquito-borne viral disease, only

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